LECTURE 10

A 3+1 Dimensional LF-Model with Spontaneous χSB

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1. INTRODUCTION

Light-front (LF) quantization provides an intuitive (physical basis!) description of hadron structure that stays close to the relevant degrees of freedom in high energy scattering: many high-energy scattering processes probe hadrons along a light-like direction, since particles at very high energies travel close to the light-cone. More detailed discussions of this main motivation for studying LF field theories can be found in Refs. [1, 2] as well as in the lecture by Stan Brodsky [3].

If the main application for LF field theory is supposed to be the phenomenology of high-energy scattering processes, must one worry about the structure the vacuum in the LF formalism? At first one might think that the answer to this question is no. However, since the naive (no 0-modes) LF vacuum is known to be trivial, one might worry, for example, whether deep-inelastic structure functions can be correctly calculated on the LF in a theory like QCD where the vacuum is known to have a nontrivial structure and where one knows that this nontrivial vacuum structure plays an important role for phenomenology.

It is well known that LF Hamiltonians allow for a richer counter-term structure [4], and spontaneous symmetry breaking in normal coordinates can manifest itself as explicit symmetry breaking counter-terms in the corresponding LF Hamiltonian. In other words, the vacuum structure is shifted from states to fields. Thus, one can account for a nontrivial vacuum structure in the renor-

malization procedure. Some immediate questions that arise in this context are

- Can a LF Hamiltonian, with a trivial vacuum, have the same "physics" (in the sense of physical spectrum or deep inelastic structure function) as an equal time Hamiltonian with nontrivial vacuum?
- What are implications for renormalization, i.e. how does one have to renormalize in order to obtain the same physics?
- What is the structure of the effective interaction for non-zero-modes

Of course, the general answer (i.e. QCD_{3+1}) is difficult to find, but above questions have been studied in simple examples ([1]):

QED/QCD_{1+1} [6, 7, 8], Yukawa$_{1+1}$ [9], scalar theories (in any number of dimensions) [10], perturbative QED/QCD_{3+1} [11] and "mean field models": Gross-Neveu/NJL-model [12].

The goal of such toy model studies is to build intuition which one can hopefully apply to QCD_{3+1} (using trial and error). However, while these models have been very useful for studying nonperturbative renormalization in 1+1 dimensional LF field theories, it is not clear to what extend these results can be generalized to sufficiently nontrivial theories in 3+1 dimensions.

2. A 3+1 DIMENSIONAL TOY MODEL

One would like to study a 3+1 dimensional model which goes beyond the mean field approximation (NJL !), but on the other hand being too ambitious results in very difficult or unsolvable models. ([2]) We decided to place the following constraints on our model:

- Most importantly, the model should be 3+1 dimensional, but we do not require full rotational invariance.
- The model should have spontaneous χSB (but not just mean field)
- Finally, it should be solvable both on the LF and using a conventional technique (to provide a reference calculation).

Given these constraints, the most simple model that we found is described by the Lagrangian

$$\mathcal{L} = \bar{\psi}_k \left[\delta^{kl} \left(i \partial\!\!\!/ - m \right) - \frac{g}{\sqrt{N_c}} \vec{\gamma}_\perp \vec{A}^{kl}_\perp \right] \psi_l - \frac{1}{2} \vec{A}^{kl}_\perp \left(\Box + \lambda^2 \right) \vec{A}^{kl}_\perp, \tag{1}$$

([1]) For a more complete list of examples and references on this topic, see Ref. [2].

([2]) For example, demanding Lorentz invariance, chiral symmetry and asymptotic freedom leaves QCD as the most simple model.

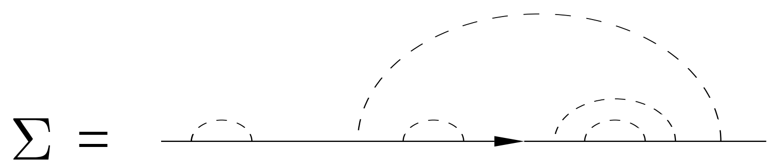

Fig. 1. — Typical Feynman diagram contributing to the fermion self-energy in the large N_C limit of the model. No crossed "gluon" lines are allowed.

where k, l are "color" indices ($N_c \rightarrow \infty$), $\perp = x, y$ and where a cutoff is imposed on the transverse momenta. A fermion mass was introduced to avoid pathologies associated with the strict $m = 0$ case. χSB can be studied by considering the $m \rightarrow 0$ limit of the model.

The reasons for this bizarre choice of model [Eq. (1)] are as follows. If one wants to study spontaneous breaking of chiral symmetry, then one needs to have a chirally invariant interaction to start with, which motivates a vector coupling between fermions and bosons. However, we restricted the vector coupling to the $\perp$ component of a vector field since otherwise one has to deal with couplings to the bad current j^- ([3]). In a gauge theory, such couplings can be avoided by choice of gauge, but we preferred not to work with a gauge theory, since this would give rise to additional complications from infrared divergences. Furthermore, we used a model with "color" degrees of freedom and considered the limit where the number of colors is infinite, because such a model is solvable, both on and off the LF. No interaction among the bosons was included because this would complicate the model too much. Finally, we used a cutoff on the transverse momenta because such a cutoff can be used both on the LF as well as in normal coordinates and therefore one can compare results from these two frameworks already for finite values of the cutoff.

3. DYSON-SCHWINGER SOLUTION OF THE MODEL

Because we are considering the limit $N_C \rightarrow \infty$, of Eq. (1), the iterated rainbow approximation (Fig. 1) for the fermion self-energy Σ becomes exact, yielding

$$\begin{aligned}
\Sigma(p^\mu) &= ig^2 \int \frac{d^4k}{(2\pi)^4} \vec{\gamma}_\perp S_F(p^\mu - k^\mu) \vec{\gamma}_\perp \frac{1}{k^2 - \lambda^2 + i\varepsilon} \\
&= \not{p}_L \Sigma_L(\vec{p}_L^2, \vec{p}_\perp^2) + \Sigma_0(\vec{p}_L^2, \vec{p}_\perp^2),
\end{aligned} \tag{2}$$

([3]) j^- is bilinear in the constrained component of the fermion field, which makes it very difficult to renormalize this component of the current in the LF framework.

with:

$$S_F^{-1} = \not{p}_L \left[1 - \Sigma_L(\vec{p}_L^2, \vec{p}_\perp^2)\right] + \not{p}_\perp - \left[m + \Sigma_0(\vec{p}_L^2, \vec{p}_\perp^2)\right] . \quad (3)$$

These equations can be solved by iteration. From the self-consistently obtained solution of the Dyson-Schwinger (DS) equation (2) one can extract the physical mass of the fermion. For sufficiently large coupling constant, the physical mass for the fermion remains finite in the limit $m \to 0$, proving the spontaneous breakdown of chiral symmetry in the model.

4. LF-SOLUTION OF THE MODEL

Since we wanted to investigate the applicability of the effective LF Hamiltonian formalism, we formulated above model without explicit zero-mode degrees of freedom. In principle, the calculation should thus be straightforward, using standard numerical techniques, such as DLCQ [13]. However, in this approach it is hard to take full advantage of the large N_C limit so that it is difficult to compare the obtained spectrum with the results from solving the DS equation. Instead, we use the following 2-step procedure to obtain a formal solution for the LF formulation

1. First, we derive a self-consistent Green's function equation which is equivalent to the DLCQ calculation. The Green's function calculation was originally derived by starting from the covariant calculation and performing k^- integrations first (throwing away zero modes in k^+). In order to convince even the skeptics that this procedure is equivalent to DLCQ, we demonstrate numerically that, for finite and fixed DLCQ parameter K, the spectrum obtained by diagonalizing the DLCQ matrix and the spectrum obtained by solving the Green's function equation self-consistently ([4]) are identical.
2. In the next step we compare the self-consistent Green's function equation with the DS equation. In order to facilitate the comparison with the LF calculation, we rewrite the DS equation (2), using a spectral representation for the fermion propagator S_F. In the resulting DS equation with the spectral density, we combine energy denominators, using Feynman parameter integral and perform the longitudinal momentum integral covariantly.

Details of this procedure can be found in Ref. [14]. The main results from the comparison between LF and DS equations are as follows

- The LF Green's function equation and the DS equation are identical (and thus have identical solutions) if and only if one introduces an additional (in addition to the self-induced inertias) counterterm to the kinetic mass term for the fermion.

([4]) Replacing integrals by finite sums in order to account for the finite DLCQ parameter K.

- For fixed transverse momentum cutoff, this additional kinetic mass term is finite.
- The value of the vertex mass in the LF Hamiltonian is the same as the value of the current mass in the DS equation.
- In the chiral limit, mass generation for the (physical) fermion occurs through the kinetic mass counter term

5. IMPLICATIONS FOR RENORMALIZATION

We have studied a 3+1 dimensional model with spontaneous breaking of chiral symmetry both in a LF framework as well as in a Dyson-Schwinger framework. Our work presents an explicit 3+1 dimensional example demonstrating that there is no conflict between chiral symmetry breaking and trivial LF vacua provided the renormalization is properly done.

The effective interaction (after integrating out 0-modes) can be summarized by a few simple terms — which are already present in the canonical Hamiltonian. The current quark mass in the covariant formulation and the "vertex mass" in the LF formulation are the same if one does not truncate the Fock space and if one uses the same cutoff on and off the LF. This is perhaps surprising, since the vertex mass multiplies the only term in the canonical Hamiltonian which explicitly breaks (LF-) chiral symmetry. Thus one might think that chiral symmetry breaking would manifest itself through a nonzero vertex mass. If one does not truncate Fock space, this is not what happens in this model! ([5]) χSB, in the sense of physical mass generation for the fermion, manifests itself through a "kinetic mass" counterterm.

Even though we determined the kinetic mass counter term by directly comparing the LF and DS calculation, several methods are conceivable which avoid reference to a non-LF calculation in order to set up the LF problem. One possible procedure would be to impose parity invariance for physical observables as a constraint [9].

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